Huaqiao University 2019 Fall Final Exam Test B

Department__________ Course___Linear Algebra___ Date___2019/12/12___

Name__________ Number__________

	1	2	3	4	5	6	total
Grade							

1 (50pts). Fill in the blanks（Write your answer on the attached sheet. 5pts/each question）

(1) Let $A=\begin{bmatrix}1&0\\-2&1\end{bmatrix}\begin{bmatrix}x&0\\0&y\end{bmatrix}\begin{bmatrix}1&0\\2&1\end{bmatrix}$. Find A^k where k is any positive integer. Your answer: ________

(2) Let A,B be 4×4 matrices. $det\,A=\frac{1}{2}, det\,B=-3, det(2AB^{-1}A^{-1})=$_____

(3) Let $A=\begin{bmatrix}2&1&2&-1\\0&0&-2&0\\-1&-4&3&-2\\-1&-2&2&-1\end{bmatrix}$. Compute the cofactors and the determinant.

$C_{23}=$____ $C_{33}=$____ and $det\,A=$____

(4) Let $S\subset\mathbb{R}^2$ be a parallelogram whose vertices are $(0,-1),(1,2),(-1,3),(-2,0)$. The area of S is ____

(5) Using Cramer's rule to solve the system of linear equations, $2sx_1+3x_2=1, x_1-sx_2=s$. We know that $x_1=\frac{det\,A_1(b)}{det\,A}$. $A=$______; $det\,A_2\,(b)=$______

(6) Let $V=\{$ differentiable real valued functions on $(0,1)\}$ and $W=\{$real valued functions on $(0,1)\}$. Let $D: V\to W$ be given by $f\mapsto\frac{df}{dx}$. The kernel of D is the set of _____ functions on $(0,1)$. $dim\,ker\,D=$____; $dim\,V=$_____

(7) A basis of P_2 is given by $1,2t,-2+4t^2$. The coordinate vector of $p(t)=2t^2+t$ is ____

(8) If A is a 6×4 matrix, what is the smallest possible dimension of Nul A? Your answer:___

(9) Let A be a 2×2 matrix with eigenvalues 2 and $\frac{1}{2}$ and corresponding eigenvectors $v_1=\begin{bmatrix}1\\1\end{bmatrix}$ and $v_2=\begin{bmatrix}-1\\1\end{bmatrix}$. Let $x_{k+1}=Ax_k$ and $x_0=\begin{bmatrix}2\\0\end{bmatrix}$. From this information we can conclude that $x_1=$_____ and the equation of the asymptotic line for large k is _____

(10) Let $A=\begin{bmatrix}1&1&0&0\\0&2&1&0\\0&0&2&1\\0&0&0&1\end{bmatrix}$. The number of eigenvalues for A is ____. List the eigenvalues with algebraic multiplicity 2: _____

2. (10pts) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

3. (10pts) Define $T: P_2 \to \mathbb{R}^2$ by $T(\boldsymbol{p}) = \begin{bmatrix} \boldsymbol{p}(1) \\ \boldsymbol{p}(0) \end{bmatrix}$.

(a) Find the image under T of $\boldsymbol{p}(t) = 1 - t - t^2$.

(b) Find a polynomial whose image under T is $\begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix}$.

(c) Find the matrix for T relative to the basis $\{-1, -t, t^2\}$.

4. (10pts) For what values of $a, b \in \mathbb{R}$ is the matrix $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ diagonalizable. For what values of $a, b \in \mathbb{R}$ is the matrix is the matrix $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ diagonalizable? Explain your answer.

5. (12pts) Let $A = \begin{bmatrix} 2 & 6 & 2 & 2 & 4 \\ -1 & -2 & -1 & 1 & -2 \\ 5 & 15 & 5 & 3 & 14 \\ 2 & 7 & 2 & 2 & 8 \end{bmatrix}$. Find a basis for Nul A, Col A, Row A respectively.

6. (8pts) Consider the discrete dynamic system $x_{k+1} = Ax_k$ where $A = \begin{bmatrix} \frac{3}{4} & p \\ \frac{1}{16} & \frac{3}{4} \end{bmatrix}$.

(a) If A has 2 positive real eigenvalues, for what values of p is the origin an attractor? And for what values of p is the origin a saddle point?

(b) When $-1 < p < 0$, is the origin an attractor or a repeller? Explain.

(c) When $p = 1$, we can conclude that $x_k = c_1\lambda_1^k v_1 + c_2\lambda_2^k v_2$. Find $\lambda_1, \lambda_2, v_1, v_2$.

(d) When $p = 1$, the trajectory starting from a general point $x_0 = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ is lying on a line l_1 and will approach another line l_2. Find the equation of l_1 and l_2.

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